Observation of Quantized Circulation in Superfluid ^{3}He-B

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We report the first observation of quantized circulation in superfluid ^{3}He-B. The apparatus consists of a straight vibrating wire immersed in liquid ^{3}He, which is cooled by a rotating nuclear demagnetization cryostat. The experiment is carried out at about 250 μK. The superfluid at this temperature is in the ballistic quasiparticle regime. Circulation around the wire is found to be stable only when it takes on the values $-h/2m_3$, 0, and $+h/2m_3$, where h is Planck's constant and m_3 is the mass of the ^{3}He atom. This experiment confirms that superfluid ^{3}He-B is a Cooper-paired superfluid with a macroscopic quantum wave function.

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The most striking characteristic of the known superfluids (superconducting electrons, superfluid ^{4}He, and superfluid ^{3}He) is their ability to maintain large mass currents without driving forces or dissipation, for long periods of time. London suggested that a "macroscopic quantum current"[1] could explain these effects. He also suggested the idea of a "macroscopic quantum state"[1] in which these systems form a condensate below their critical temperature, where a large number of particles occupy a single quantum state.

These ideas were united in a single mathematical description with the postulate[2,3] that a superfluid can be described by a macroscopic quantum-mechanical wave function $\Psi=|\Psi|e^{i\Phi}$. This wave function should be single valued, which immediately implies the quantization of circulation in a neutral superfluid[2,4] and of magnetic flux[5,6] in a charged superfluid. It is not, however, possible to predict the magnitude of the quantum unit for any of these systems without a microscopic theory.

In the case of superfluid ^{4}He a complete microscopic theory does not exist. Superfluidity is, however, thought to be related to Bose-Einstein condensation in which large numbers of single ^{4}He atoms condense into the lowest quantum state of the system. This led to the postulate that circulation κ is quantized in units of h/m_4 (Refs. 2 and 4) (where m_4 is the mass of the ^{4}He atom). Confirmation of this postulate[7–9] lent strong support to the idea of the macroscopic quantum wave function.

For the case of superconducting electrons, BCS theory[10] indicates that the condensate is made up of bound pairs of electrons called Cooper pairs[11] and gives microscopic validity to the macroscopic wave function. This led to the prediction[12] that flux should be quantized in units of $h/2e$. The experimental demonstration of the quantization of flux in these units[13,14] was proof of the existence of these pairs and the confirmation of both the BCS theory and the macroscopic wave function.

The discovery[15] of new phases of liquid ^{3}He was soon followed by their theoretical description as Cooper-paired, neutral BCS superfluids.[16–18] This description leads to the conclusion that circulation should be quantized in units of $h/2m_3$, since the mass of the Cooper pairs making up the condensate is $2m_3$. Although there is much data consistent with this BCS theory, there has, until now, been no single experiment which unambiguously[19] reveals the existence of a macroscopic wave function, quantized circulation, or Cooper pairs in superfluid ^{3}He.

Our experimental method for measurement of superfluid circulation is essentially the vibrating-wire technique which was used for ^{4}He by Vinen,[8] and later by Zimmermann and co-workers.[9] For our investigations of superfluid ^{3}He-B, a superconducting NbTi wire of diameter 16 μm is mounted along the axis of a 50-mm-long, 2.8-mm-inner-diameter brass cylinder containing ^{3}He. The tension in the wire is set by gluing it in position with epoxy while a small weight hangs from one end. The cylinder is connected, through a small ($\sim$1-mm-diam) opening, to the heat exchanger of a rotating nuclear demagnetization cryostat.[20] The axis of the wire is parallel to the rotation axis and perpendicular to a 50-mT magnetic field. This field is provided by two crossed Helmholtz pairs which enable the field direction to be rotated through 360°.

In order to make a measurement of the fluid circulation around the wire, a pulse of current is passed through it for about 1 ms. This creates a magnetic force impulse which pulls the wire to one side. After the pulse, the wire vibrates freely at its fundamental frequency near 347 Hz. Initially the vibration is perpendicular to the magnetic field. If there is circulation around the wire, the plane of its motion precesses, and thus the component of the wire's motion along any given direction shows a beat pattern. The emf generated between the ends of the wire as the wire vibrates in the applied field is proportional to the component of the wire's velocity perpendicular to the field. As the velocity vector precesses, the emf also shows a beat pattern.

In practice, the cross section of the wire is not round and there are asymmetries in the supports at each end.

These imperfections cause the emf to show beats even in the absence of circulation. If t_0 is the beat period due to these asymmetries, then t_c, the precession period due to circulation, is related[8,9] to the observed beat period t_1 by

$$(1/t_c)^2=(1/t_1)^2-(1/t_0)^2. \quad (1)$$

The circulation κ around the vibrating wire is[8,9]

$$\kappa=2\pi\mu/\rho_s t_c , \quad (2)$$

where ρ_s is the superfluid density, and μ is the mass per unit length of the wire μ_w plus a correction for the inertia of the displaced fluid.[9]

The vibrating-wire cell was initially tested in rotating superfluid ^{4}He. Here quantized circulation states were observed. The accuracy of these measurements is limited by our knowledge of μ_w. However, the quantum of circulation was measured to be h/m_4 to within about 5% using a value for μ_w determined from the manufacturer's specifications for the wire. We use the ^{4}He measurement as a calibration of the device on the assumption that the quantum of circulation in ^{4}He is exactly equal to h/m_4. This calibration indicates that $\mu_w=(1.53 \pm 0.08)\times 10^{-6}$ kg m^{-1}.

A single quantum of circulation in superfluid ^{3}He-B can be measured precisely only if the damping of the wire's vibration is sufficiently small that several beats can occur before the amplitude becomes too small to measure. Since the quasiparticle damping is a decreasing function of the temperature T, this condition places a practical upper limit of about $T/T_c=0.2$ on the ^{3}He temperature, where T_c is the temperature at which liquid ^{3}He becomes superfluid. In this temperature range, the superfluid is in the ballistic quasiparticle regime.[21] To generate the circulation the cryostat must rotate at speeds of order 1 rad s^{-1} while remaining at these temperatures. A rotating millikelvin cryostat with these capabilities has recently been completed at Berkeley.[20] The experiment was carried out in superfluid ^{3}He-B at a pressure of 17.8 bars and at temperatures between 250 and 400 μK.

Figure 1 shows the emf induced across the vibrating wire as a function of time during a free decay of oscillations. On the time scale shown the individual oscillations cannot be resolved. The envelope of these oscillations is seen to be a combination of low-frequency beats, due to the precession of the wire's motion, and the decay of the amplitude of its vibration. The Q of these decays ranges from 500 to 5000 in the temperature range of this experiment. Figure 1 shows a typical signal obtained when the ^{3}He is cooled into the superfluid phase without rotation. In this case the observed beat period t_0 (the time between two successive zeros of the beat pattern) is due only to the wire's asymmetry. When the rotation speed of the cryostat, Ω, has been ramped from 0 up to about 1 rad s^{-1} and then down to 0 again the beat period t_1 is noticeably shorter because of trapped circulation around the wire.

These free-decay signals, which are digitally stored, are subsequently analyzed by passing the reconstructed signal through a lock-in amplifier whose reference frequency is 347 Hz. The resulting envelope of the amplitude of oscillations for the signals where $\Omega=0$ and $\Omega=1$ rad s^{-1} are shown superimposed in Fig. 2. The measured difference in beat periods is used with Eqs. (1) and (2) to calculate the circulation around the wire.

A typical run of the experiment proceeds as follows. The sample is cooled at rest into the superfluid.[22] After the demagnetization cooling is complete and the temperature equilibrates, the wire is "plucked" every 30 s to determine its beat period and thus the circulation around it. The rotation speed Ω of the cryostat is ramped up at

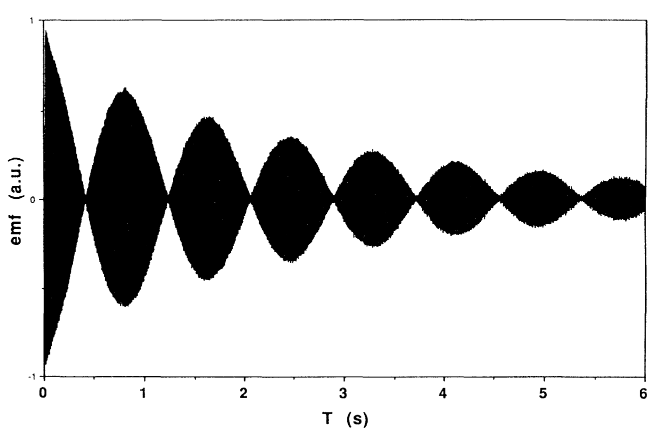

FIG. 1. Emf from a free decay of oscillations when the superfluid has not been rotated.

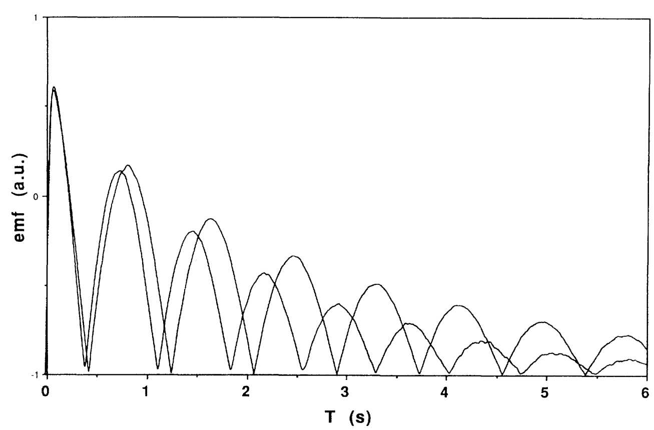

FIG. 2. The amplitude envelopes of two signals as a function of time. In one $\Omega_{max}=0$ and in the second $\Omega_{max}=1$ rad s^{-1}. The difference in decay times is due to slightly different temperatures and is not related to the circulation difference.

an acceleration α of about 0.003 rad s^{-2} to a speed Ω_{max} and then it is ramped down to $\Omega=0$ at the same rate.

After each ramp is finished, the circulation κ around the wire is measured at $\Omega=0$. We continue the measurement of κ for between 5 and 10 min in order to investigate the stability of the trapped state. Positive and negative Ω_{max} correspond to clockwise and anticlockwise rotation of the cryostat, respectively. Figure 3 shows the results of this series of beat-period measurements. Figure 4 shows the calculated values of the average circulation, in units of $h/2m_3$, as a function of Ω_{max}. Only the points calculated from stable values of t_1 are shown. The form of Eq. (2) is such that, in the zero-circulation state, variations in the beat period due to random electronic noise result in an offset to the magnitude of values of calculated circulation. In Fig. 4 we have treated this nonphysical rectification and offset by subtracting from the calculated circulation the mean circulation which would appear as a result of the measured noise. This subtraction is made on the assumption that all the data shown for $|\Omega_{max}| \lesssim 0.5$ rad s^{-1} represent a situation dominated by electronic noise and where real departures of the circulation from zero are negligible.[23] The corrections to the data for the states where $|\Omega_{max}| \gtrsim 0.5$ rad s^{-1} are insignificant. The measurement of the magnitude of the quantum of circulation comes directly from the t_1 data and is independent of these noise corrections.

The sign of κ can be distinguished by observing changes to the beat pattern after a rapid change is made to the magnetic field direction. This method was developed by Zimmermann and co-workers.[9] Using this technique we find that changing the sign of Ω results in a sign change in κ for all checked circulation states. In plotting Fig. 4 we have assumed that all trapped circulations, near $|\kappa| = h/2m_3$, have the same sign as Ω_{max}.

In Fig. 4 three different regimes can be identified.

(1) $\Omega_{max} \lesssim 0.5$ rad s^{-1}: The circulation is stable and zero.

(2) $\Omega_{max} \approx 0.5$ rad s^{-1}: The circulation is unstable and has been observed to fluctuate for periods of up to several hours. The regions between the arrows in Fig. 4 extend over the ranges of Ω_{max} where the circulation is unstable. Since the figure only displays stable levels there are no points within these regions.

(3) $\Omega_{max} \gtrsim 0.5$ rad s^{-1}: The circulation is stable to within the electronic noise. States of trapped circulation of this magnitude have been observed to remain unchanged for at least 14 h.

For these trapped-circulation states the calculated magnitude of κ, in units of $h/2m_3$, is 1.05 ± 0.05. Here the error is systematic, originating in the uncertainty of μ_w based on our ^{4}He data.

In order to investigate the state with quantum number $n=+1$, after an experiment which traps the $n=-1$ state on the wire, the ^{3}He in the cell is warmed above the superfluid critical temperature. This destruction of the superfluidity removes both free vortices from the cell and circulation from around the wire. If this is not done, there are strong hysteresis effects in the observed dependence of κ on Ω. No states with n of magnitude greater than 1 were observed in superfluid ^{3}He-B where the maximum rate of rotation attempted was $\Omega \sim 3$ rad s^{-1}. In contrast, states of all quantum numbers from 0 to 3 were observed with the same wire, and in the same range of Ω, when it was surrounded by superfluid ^{4}He. We believe that this difference results from the different core

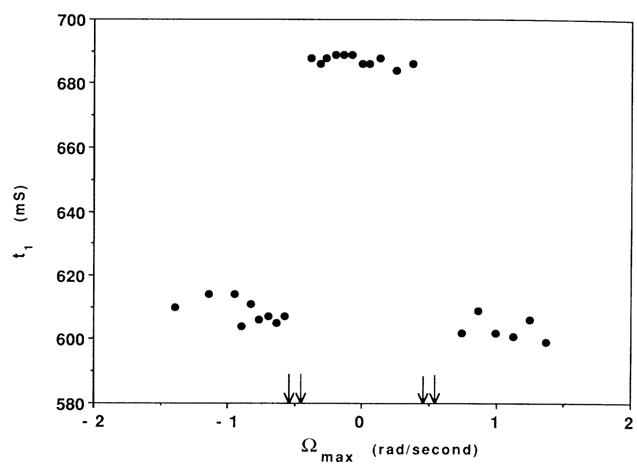

FIG. 3. The average beat period measured as a function of the maximum rotation speed to which the sample has been subjected. The average is taken over the periods measured during the time that the cryostat is stopped at the end of each ramp. The beat period is unstable in the regions between the two sets of arrows.

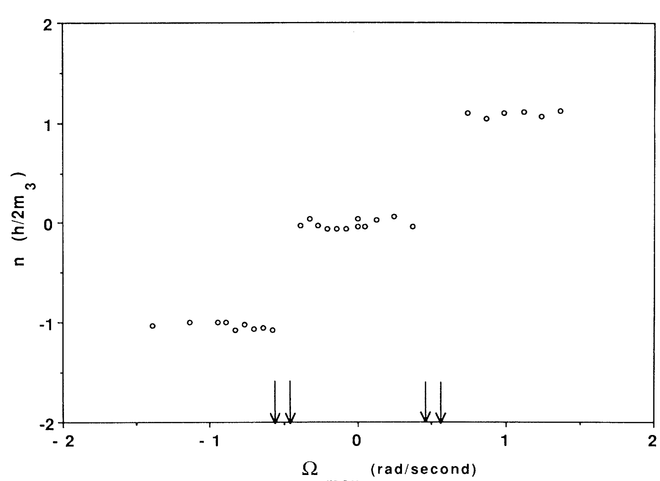

FIG. 4. The calculated value of the circulation, when the noise rectification is taken into account, as a function of Ω_{max}. The circulation is unstable in the regions between the two sets of arrows.

sizes of singular vortices in these two superfluids. The vortex core is thought to be only about 0.2 nm in diameter for ^{4}He, whereas for ^{3}He-B, at 17.8 bars, it is about 100 nm.[24] This means that the kinetic energy of a ^{4}He vortex is higher than that of a singular ^{3}He-B vortex. Thus the free-energy difference between a vortex and a state of quantized circulation around a 16 μm wire is larger for ^{4}He than for ^{3}He-B. It is therefore energetically more favorable to have higher quantum numbers trapped in superfluid ^{4}He.

In conclusion, we have demonstrated that stable trapped circulation around a small-diameter wire in superfluid ^{3}He-B is quantized in units of $h/2m_3$. This confirms the existence of Cooper pairs and of the macroscopic quantum wave function in this superfluid. Three quantum states, $n=+1$, $n=0$, and $n=-1$, have been observed.

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[1]F. London, *Superfluids* (Wiley, New York, 1954), Vol. 2, pp. 42, 59, and 142.

[2]R. P. Feynman, in *Progress in Low Temperature Physics,* edited by C. T. Gorter (North-Holland, Amsterdam, 1955), Vol. 1, Chap. 2.

[3]V. L. Ginzburg and L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. **34**, 1240 (1958) [Sov. Phys. JETP **7**, 858 (1958)].

[4]L. Onsager, Nuovo Cimento **6**, Suppl. 2, 249 (1949).

[5]F. London, *Superfluids* (Wiley, New York, 1954), Vol. 1, p. 152.

[6]L. Onsager, in *Proceedings of the International Conference on Theoretical Physics, Kyoto and Tokyo, September 1953* (Science Council of Japan, Tokyo, 1954), p. 935.

[7]G. W. Rayfield and F. Reif, Phys. Rev. **136**, 1194 (1964).

[8]W. F. Vinen, Proc. Roy. Soc. London A **260**, 218 (1961).

[9]S. C. Whitmore and W. Zimmermann, Jr., Phys. Rev. **166**, 181 (1968); P. W. Karn, D. R. Starks, and W. Zimmermann, Jr., Phys. Rev. B **21**, 1797 (1980).

[10]J. Bardeen, L. N. Cooper, and J. R. Schreiffer, Phys. Rev. **108**, 1175 (1957).

[11]L. N. Cooper, Phys. Rev. **104**, 1189 (1957).

[12]N. Byers and C. N. Yang, Phys. Rev. Lett. **7**, 46 (1961).

[13]B. S. Deaver and W. M. Fairbank, Phys. Rev. Lett. **7**, 43 (1961).

[14]R. Doll and M. Nabauer, Phys. Rev. Lett. **7**, 51 (1961).

[15]D. D. Osheroff, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. **28**, 885 (1972).

[16]R. Balian and N. R. Werthamer, Phys. Rev. **131**, 1553 (1963).

[17]P. W. Anderson and P. Morel, Phys. Rev. **123**, 1911 (1961); P. W. Anderson and W. F. Brinkman, Phys. Rev. Lett. **30**, 1108 (1973).

[18]A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).

[19]The periodicity in the response characteristics of the two-hole superfluid Helmholtz oscillator can be interpreted as being evidence for quantized circulation states in superfluid ^{3}He-B; O. Avenel and E. Varoquaux, Jpn. J. Appl. Phys. **26**, Suppl. 26-3, 1798 (1987); Phys. Rev. Lett. **60**, 416 (1988).

[20]J. Close, R. Zieve, J. C. Davis, and R. E. Packard, Physica (Amsterdam) **165 & 166B**, 57 (1990).

[21]A. M. Guenault, V. Keith, C. J. Kennedy, S. G. Mussett, and G. R. Pickett, J. Low Temp. Phys. **62**, 511 (1986).

[22]Recently it has been suggested by A. Corrada-Emmanuel (to be published) that the circulation state at $\Omega=0$ is not necessarily an integral multiple of $h/2m_3$ if the superfluid is created under rotation. This experiment does not test that prediction.

[23]A more detailed explanation of the effect of electronic noise will be given in a future publication.

[24]M. M. Salomaa and G. E. Volovik, Rev. Mod. Phys. **59**, 585 (1987).