dc Supercurrents from Resonant Mixing of Josephson Oscillations in a ^{3}He Weak Link

R. W. Simmonds, A. Loshak, A. Marchenkov, S. Backhaus,* S. Pereversev,†
S. Vitale,‡ J. C. Davis, and R. E. Packard
Department of Physics, University of California, Berkeley, California 94720
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We have discovered that the dc mass current through a superfluid ^{3}He weak link is substantially increased when the Josephson frequency matches the resonant frequency of a coupled mechanical oscillator. The phenomenon is the result of homodyne mixing between the Josephson oscillations and the oscillating pressure field associated with the resonant system. The measured sizes of the current enhancements are in excellent agreement with calculations based on this homodyne model. Similar observations in superconducting junctions, in which microwave radiation changes the dc electronic current, were used for the first confirmation of the dynamics of the superconducting Josephson effect. [S0031-9007(98)06801-X]

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Recent experiments have shown that an array of submicron sized apertures, connecting two reservoirs of superfluid ^{3}He, exhibits properties characteristic of a quantum weak link [1,2]. In particular, the mass current I is observed to be proportional to the sine of the quantum phase difference across the link ϕ, i.e.,

$$I(\phi) = I_c \sin(\phi). \tag{1}$$

Equation (1) is analogous to the superconducting dc-Josephson equation. The quantum phase difference ϕ evolves according to [3]

$$\phi(t) = -\int_0^t \frac{2m_3}{\rho \hbar} P(t')\, dt', \tag{2}$$

where ρ is the density of the liquid, $2m_3$ is the mass of a Cooper pair of ^{3}He atoms, and P is the pressure difference across the weak link. When a constant pressure difference P_0 is applied across the array, one observes mass currents oscillating at the superfluid Josephson frequency f_j where

$$f_j = \frac{2m_3}{\rho h} P_0 = 183.7 \text{ kHz/Pa} \tag{3}$$

and h is Planck's constant.

These experimental results lead one to ask if dynamical Josephson effects in ^{3}He, analogous to those seen in superconducting weak links, might be observable. The first dynamical phenomenon to be observed for superconducting weak links was the Shapiro effect [4] in which microwave radiation influences the dc current through a superconducting junction. An important related phenomenon is the Fiske effect [5], where interaction of the Josephson oscillations with internal electrodynamic resonances in the junction causes current "spikes." Another related effect is resonant dc current enhancement spikes in a voltage biased superconducting Josephson junction, when the emitted microwave radiation mixes with the response of a coupled microwave resonator [6]. This is the closest analogy to the results reported in this Letter which describes an experiment that reveals an increase in the dc mass current when the ^{3}He Josephson frequency matches that of a coupled mechanical oscillator.

Our experimental apparatus (which is shown schematically in Fig. 1 and described more fully in Ref. [1]) contains a flat, hollow, cylindrical cell, formed by gluing a flexible plastic membrane (stiffness $k_1 \approx 4.7 \times 10^3$ N/m) to the top of a 140-μm-thick plastic washer whose inner diameter is 12.7 mm. A stiffer membrane (stiffness $k_2 \approx 11 \times 10^3$ N/m) is glued to the bottom of this washer. This membrane contains the weak link which consists of a 65×65 array of 100 nm holes spaced 3000 nm apart, etched through a 50 nm thick SiN membrane. Previous work demonstrated that such an array behaves coherently like a single weak link whose critical

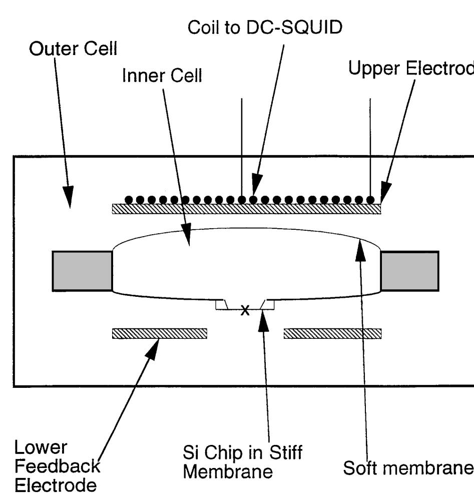

FIG. 1. Schematic diagram of the experimental apparatus. A flat, ^{3}He filled, cylinder forms the inner cell which is contained in an outer cell of superfluid ^{3}He. A stiff membrane is glued to the bottom of the inner cell and contains the weak link array. The upper, softer membrane, which is coated with a superconducting film, faces a SQUID-based displacement transducer and a planar electrode. The lower stiffer membrane is also metal coated and faces a different planar electrode.

current is approximately 65^2 times greater than that in the individual apertures [2]. The upper, softer membrane, which is coated with a superconducting film, faces a SQUID-based displacement transducer and a planar electrode. The lower stiffer membrane is also metal coated and faces a different planar electrode which can pull on it via the application of electrostatic potentials. The cell is immersed in a container of superfluid He^3, which is in contact with a nuclear demagnetization refrigerator. After initially filling the outer container, the inside of the cell fills through the aperture array. Thermometry is based on the NMR susceptibility of ^{195}Pt. The experiment is performed at zero ambient pressure where the temperature-dependent coherence length $\xi(T)$ is comparable to the aperture dimensions: $\xi(T) = (65\text{ nm})/(1 - T/T_c)^{1/2}$.

To carry out these measurements we have developed a unique new system that permits us to drive flow through the aperture array at a fixed pressure head [7]. This is accomplished by using a feedback loop which applies an electrostatic force to the stiffer membrane. The force causes the membrane to move so that the pressure difference across the weak link, determined by the position of the softer membrane, is kept fixed at a prechosen value P. The force grows linearly in time at a rate proportional to the dc mass-current I which is associated with the particular pressure head P. This technique provides the neutral fluid equivalent of a "voltage bias" in a charged system.

The feedback control system allows us to measure the relation between mass current and pressure head. This I-P curve is analogous to the I-V curve of a charge carrying device. To generate the current-pressure curve we step though the pressure domain, recording the dc flow from the feedback signal for times of order 1–30 sec. At each pressure P we measure the corresponding Josephson frequency f_j, by observing the frequency of oscillations of the soft diaphragm in response to the Josephson current oscillations in the weak link. This provides an absolute *in situ* calibration of the pressure P, from Eq. (3). The displacement of the soft membrane is used as the feedback parameter instead of f_j because it is a direct high-accuracy signal, proportional to P, and needs no further processing. We record the mass current associated with P and $-P$ before stepping to the next pressure. A complete sweep from 0 to ± 20 mPa takes on the order of 90 min. During this time the temperature remains constant due to the thermal mass of the nuclear stage. Additional I-P curves are taken by changing the temperature and repeating the measurement cycle.

Figure 2(a) shows a typical I-P curve, where P is plotted in units of frequency from Eq. (3). These I-P curves are symmetric about the pressure origin. A slowly varying background current [8] has been subtracted to clarify the position of the current amplification peaks. Our cell exhibits several distinct resonant mechanical modes. In addition to the so-called pendulum mode [9], which involves direct flow through the apertures, there are other modes involving the membranes' motion and coupling to fluid flow in the surrounding space. The potential energy term for each resonance involves a particular bending of the membrane, and the inertial term involves the kinetic inductance of the flow paths on the outside of the inner cell. We believe that the lowest mode results from the in-phase azimuthally symmetric motion of the two membranes, while the higher modes are associated with azimuthally asymmetric modes of the soft membrane. We experimentally determine the normal modes of the cell by measuring the response function of the soft membrane when an electrostatic force is applied to the stiff membrane. Figure 2(b) shows a typical cell spectrum. The modes range up to several kilohertz and, for a given mode, the frequency varies with temperature proportional [10] to $(1 - T/T_c)^{1/2}$ which is expected if the mode frequencies are proportional to the square root of superfluid fraction. Comparison between Figs. 2(a) and 2(b) indicates that each current peak is associated with a resonance of the cell.

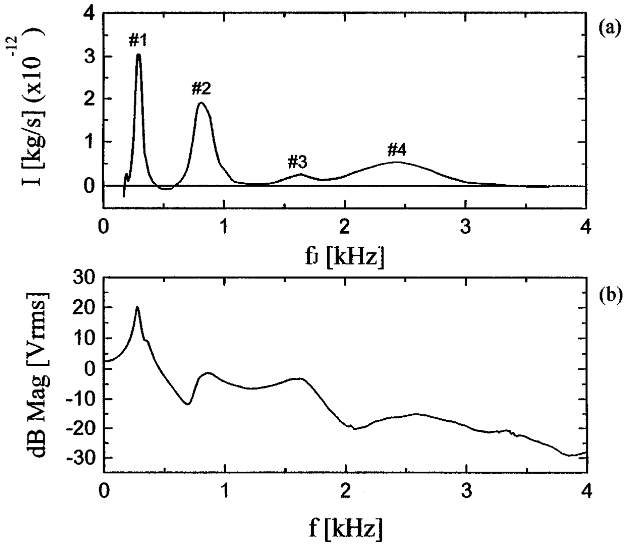

FIG. 2. (a) A typical current-pressure curve, after the slowly varying background has been removed to clarify the position of the peaks. This I-P was measured at $T/T_c = 0.572$. (b) The response of the upper membrane to an oscillating force applied to the lower stiff membrane, also measured at $T/T_c = 0.572$. The vertical scale is logarithmic.

We explain the presence of the current peaks by a perturbative model similar to that used for a superconducting Josephson junction exposed to an applied microwave field [11]. The constant applied pressure P_0 drives an oscillating Josephson current which in turn can drive a cell resonance. The resonant motion alters the applied pressure across the aperture with an additive small harmonic term. This term, when mixed with the original oscillation, results in a dc component to the current.

For instance, in the first resonant mode of the cell, the two membranes oscillate together (in phase) by displacing the fluid outside the cell. At resonance, the current, oscillating with magnitude I_c at the Josephson frequency $\omega_j = 2m_3P_0/\rho\hbar$, creates an oscillating displacement of the soft membrane. This in turn rings up the cell resonance, which, when its equilibrium amplitude is achieved, results in an oscillation pressure difference given by

$$\delta P = I_c \frac{Q}{\omega_j} \frac{k_2}{\rho A_2^2} \alpha \sin(\omega_j t), \quad (4)$$

where Q is the quality factor of the resonance and $\alpha = (k_2/A_2^2)/(k_1/A_1^2 + k_2/A_2^2)$. Here A_1 and A_2 are the areas of the softer membrane and of the stiffer one, respectively.

This oscillating pressure corresponds to an oscillating phase difference term

$$\delta\phi = -\int \delta P \frac{2m_3}{\rho\hbar} dt. \quad (5)$$

By adding this term in the expression for the current $I = I_c \sin(\omega_j t + \delta\phi) \approx I_c \sin(\omega_j t) + I_c \delta\phi \cos(\omega_j t)$, the current acquires a dc component:

$$\Delta I_{\rm dc} = \frac{1}{2} \frac{2m_3}{\hbar} \frac{k_2}{\rho^2 A_2^2} \alpha \frac{I_c^2 Q}{\omega_j}. \quad (6)$$

For other resonances this general form is preserved with an appropriate different value for α.

This model predicts a current peak which is the result of down conversion to zero frequency resulting from the homodyne mixing between the Josephson oscillation and the oscillating pressure due to cell resonance. The peak occurs when the Josephson frequency matches that of the cell resonance and the current enhancement is given by Eq. (6).

We observe current peaks in the I-P characteristic over a range of temperatures. The frequencies at which the cell resonances occur are temperature dependent because the inertia of the oscillator varies with the superfluid density. At a given temperature we determine the relation between the pressure at which a current peak occurs, and the cell resonance frequency. For flow at fixed pressure we directly detect the Josephson oscillations and therefore have an *in situ* calibration between pressure and quantum oscillation frequency through Eq. (3). Thus, from the measured pressure we know the Josephson frequency associated with a particular current peak, and match that frequency with the closest cell resonance frequency. Figure 3 shows the Josephson frequency at which a current enhancement peak occurs plotted against the resonant frequency of the associated cell resonance. Data for the four lowest frequency mechanical resonances are shown at a wide range of temperatures. The straight line, with a slope of one, is the experimental confirmation that the Josephson frequency associated with the current peak matches the cell resonance frequency.

It is of historical interest to note that the superconductivity experiment, which is analogous to the one reported here, was carried out before the direct observation of the

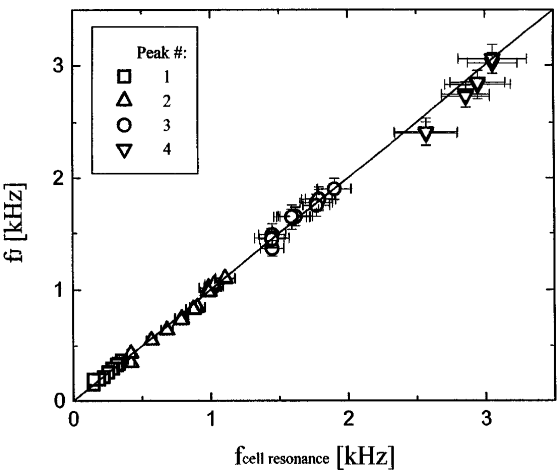

FIG. 3. The relation between the Josephson frequency at a current enhancement peak and the frequency of the closest cell resonance. The straight line has unit slope implying that the current peaks occur when the Josephson frequency matches that of a resonance.

Josephson frequency in that system. Thus, if we did not *a priori* know the Josephson frequency, the vertical axis in Fig. 3 would be the pressure head across the weak link, determined directly by knowing the position of the membrane, its spring constant k_1, and the area of the membrane. Then, the straight line in Fig. 3 would constitute a verification of the Josephson frequency relation, analogous to the dc current method, via the Shapiro steps, first used in superconductors [4].

Equation (6) gives the predicted relationship between the current enhancement $\Delta I_{\rm dc}$, the Q, the critical current I_c, and the frequency of the mechanical oscillations, in the temperature regime where the current-phase relation is sinelike. At each temperature we can directly determine the critical current of the weak link array by determining the current phase relation [2]. The Q and resonance frequency of each mode are directly measured from the cell's resonance response function. Figure 4 displays the plot of $\Delta I_{\rm dc}$ versus $I_c^2 Q/\omega^2$ which according to Eq. (6) should be a straight line at the higher temperatures where $I \propto \sin(\phi)$. At lower temperatures, where the $I(\phi)$ is no longer sinelike [2], $\Delta I_{\rm dc}$ should no longer follow this line. This is confirmed by the departure of the measured values of $\Delta I_{\rm dc}$ from the theoretical prediction which is shown as the solid line in Fig. 4. The straight line fit to the data which has ordinate value less than two (this corresponds to data with temperatures greater than $T/T_c = 0.64$) has a slope of $(3.7 \pm 0.3) \times 10^{15}$ (SI). This agrees well with the prediction from Eq. (6), using no free parameters, of 3.6×10^{15} (SI).

The quantitative agreement of the experiment with all aspects of the model provides strong confirmation of the resonant homodyne mechanism. This means that the results reported here confirm a deep analogy, which has been the object of a number of experimental searches [12]

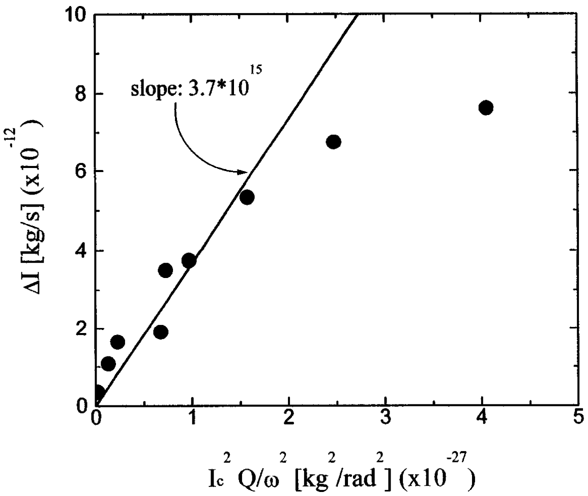

FIG. 4. Measured magnitude of the current enhancement ΔI_{dc} versus $I_c^2 Q/\omega^2$ for the lowest frequency peak. For data which has an ordinate value less than 2 [i.e., for which $T/T_c > 0.64$ where the $I(\phi)$ is known to be sinelike] the points fit a line which has a slope of $(3.7 \pm 0.3) \times 10^{15}$ (SI). This agrees well with the prediction for a sinelike $I(\phi)$, which is shown as a solid line, which has slope 3.6×10^{15} (SI), from Eq. (6). At lower temperatures (on the right) the data depart from the prediction as expected.

over the years, between superfluid Josephson dynamics and the well known dynamics of superconducting Josephson junctions.

In addition to giving us a better understanding of the dynamics of ^{3}He weak links, there are several important implications of the dc current amplification peaks. For example, since the dc current at the peak scales with I_c^2, the peak current could be used as the physical observable in a quantum interference device such as a ^{3}He dc-SQUID [13]. Also, a feedback mechanism which maintains the ΔI at its maximum value may provide a very precise pressure standard via Eq. (3). In addition, the resonance mixing phenomenon suggests that a ^{3}He Josephson weak link could serve as a more general purpose mixer which might be useful for instruments designed to detect very weak mechanical and acoustic oscillations.

In conclusion, we have discovered self-induced dc mass current enhancements caused by the resonant excitation of mechanical modes of the cell by the Josephson oscillations in a weak link array. The observed relationship between the pressure position of the current peaks and the resonance frequencies of the cell, in addition to the predicted magnitude of the peaks, provide confirmation of a homodyne mechanism. The result of this experiment suggests that this type of weak link array will be capable of exhibiting other dynamical Josephson weak link phenomena.

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*Present address: Los Alamos National Laboratory, Los Alamos, NM.

†Present address: Physics Department, State University of New York at Stony Brook, Stony Brook, NY.

‡Permanent address: Department of Physics, University of Trento, Trento, Italy.

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[10] Although the geometry of the cell is quite complicated, a rough estimate of the lowest normal mode frequencies can be made by knowing the stiffness of the membranes and the geometry of the surrounding flow paths. These estimates predict modes whose frequencies are in relatively good agreement with the experimental observations.

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