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Capacitive Generation and Detection of Third Sound Resonances in Saturated Superfluid ^{4}He Films

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We report on the operation of a third sound resonator for use with saturated superfluid helium films. Measurements of superfluid ^{4}He third sound velocities as a function of film thickness are described for films ranging in thickness from 26 nm to 85 nm. The film resides on the gold-covered surface of a 38 mm diameter copper disk, which has been machined with a diamond-tipped cutting tool, and has an rms roughness of about 10 nm.

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1. INTRODUCTION

Third sound is the surface wave on a film of superfluid helium in which the superfluid component oscillates parallel to the substrate, while the normal-fluid component remains clamped to the substrate due to its viscosity. The motion of the superfluid creates local oscillations in the thickness, the superfluid density, and the temperature of the film.

Since its prediction by Atkins[1], and its subsequent experimental discovery by Everitt, Atkins, and Denenstein[2], third sound has provided a valuable tool for probing films of superfluid ^{4}He. For example, it has been used in studies of the critical velocity of a flowing film[3], of the Kosterlitz-Thouless phase transition in 2D films[4], of the layered nature of ^{3}He-^{4}He films[5], of the van der Waals interaction[6], of surface non-wetting by helium[7], of vorticity in thin films[8], of bound states of ^{3}He atoms[9], and of the vortex core size in very thin films[10].

Third sound's usefulness as a probe of ^{4}He films has long suggested its potential value for studies of films of superfluid ^{3}He. The goal of this work was the demonstration of an apparatus capable of searching for third sound in saturated films of superfluid ^{3}He.

We have therefore designed an apparatus for the capacitive generation and detection of third sound resonances, in a saturated film, and in a cell suitable for refrigeration below 1 mK. Certain design concerns arise for saturated film resonators which are not present when working with unsaturated films. For example, a saturated film will spontaneously fill the space between two capacitor plates if the distance between them is set by a spacer which touches both electrodes. Also, since the Kapitza resistance to a ^{3}He film is high, and since we generally avoid delivering heat to a sub-mK sample, we have chosen to use a capacitive generation and detection system rather than a thermal excitation system. In this paper we report the use of this apparatus to measure third sound resonances in saturated ^{4}He films.

2. EXPERIMENTAL CELL

The experimental resonator is shown schematically in Fig. 1. It consists of a horizontal copper disk with radius R = 19.1 mm, whose top surface is the substrate for the helium film. Two concentric metal plates are suspended by a supporting structure (not shown), and are positioned 15 μm above this substrate but nowhere touch it. One of the plates is a disk with radius 10.1 mm and the other is an annular ring with inner radius 13.6 mm and outer radius 17.5 mm. Each of these plates forms a parallel plate capacitor with the substrate. Applying a potential difference between the ring, or drive plate, and the substrate produces a force on the dielectric helium film. This force, which tends to thicken the film locally, is the excitation method for the third sound. When a voltage applied to the drive plate produces a force at the frequency of one of the modes of the resonator, the resonance can be observed as a changing capacitance between the inner electrode, or detection plate, and the substrate. A 2 mm wide grounded metal ring between the drive and detection plates acts as an electrostatic shield.

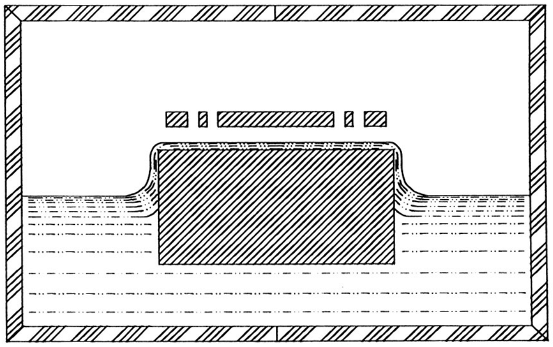

Fig. 1. Schematic of the third sound resonator.

The substrate is a copper disk whose top surface has been machined with a diamond-tipped cutting tool and then covered with a flat gold coating between 0.1 and 0.2 μm thick[11]. The gold coating prevents the formation of oxides on the substrate. AFM images of this surface showed it to have an rms roughness of about 10 nm. The coating extends to within 0.5 mm of the circular edge, which is rounded with a radius of curvature of 0.3 mm. In contrast to the substrate surface, the rounded edge is rough, having scratches on the 10 μm scale. Our results, described below, show that this rim imposes a reflective boundary condition which allows for third sound standing waves on the ^{4}He film.

The thickness of the film is varied by adding more bulk liquid helium to the cell through a fill line below the resonator. This reduces the height h from the free surface of the bulk liquid to the substrate, which causes the film to thicken in order to maintain chemical equilibrium. A cylindrical coaxial capacitor (not shown) is used to measure h. For this experiment, the cell was submerged in a liquid helium bath, which was pumped down to a base temperature of 1.4 K. The temperature was determined by measuring the vapor pressure of the helium in the bath.

3. MODEL

From the hydrodynamic equations of motion, one can obtain[12] the velocity c_3 of third sound, $c_3^2 = (\rho_s/\rho) f\, d$, where ρ_s is the superfluid density of the film, ρ is its total density, f is the van der Waals restoring force per unit mass exerted on the film's surface by the substrate, and d is the film's thickness. (Here we have neglected a small correction proportional to the temperature of the film.) By assuming a power law dependence for the van der Waals potential, it can be shown that this is equivalent to

$$c_3^2 = (\rho_s/\rho)\, n\, g\, h \tag{1}$$

where n is the exponent of the van der Waals potential (n = 4 for thick films), and g is the gravitational acceleration.[2]

The shape of a third sound standing wave Ψ, or equivalently the change in the film thickness from its equilibrium value, will be given by the solution of the wave equation in cylindrical coordinates (r, ϕ)

$$\Psi(r,\phi,t) = \sum_{m,k} \psi_{mk} J_m(kr) e^{im\phi} e^{-i\omega_k t} \tag{2}$$

where ψ_{mk} are constants, $J_m(kr)$ is the Bessel function of order m, and $\omega_k = c_3 k$ is the dispersion relation. To the extent that the resonator is azimuthally symmetric, only the $m = 0$ modes survive. Furthermore, our data supports the hypothesis that the circular edge of the substrate at $r = R$ imposes a fixed boundary condition on the surface displacement, requiring $J_o(kR) = 0$. This condition is satisfied for $k_n = x_{on}/R$ where x_{on} is the nth zero of the $J_o(x)$ Bessel function (x_{on} = 2.405, 5.520, 8.654, etc.). Thus, the resonant frequencies for third sound modes on the substrate are related to the third sound velocity by

$$\nu_n = (x_{on}/2\pi R)\,c_3 \tag{3}$$

4. MEASUREMENTS

To measure the resonant frequencies of the third sound modes, we apply a potential difference $V = V_{dc} + V_{ac}\sin(2\pi\nu t)$ between the drive plate and substrate, where $V_{dc} >> V_{ac}$ and the frequency ν is slowly swept from 0 to 100 Hz. Due to the dielectric constant $\varepsilon_4 = 1.0572$ of the liquid ^{4}He, this produces a varying drive force on the film proportional to $2\,V_{dc}\,V_{ac}\sin(2\pi\nu t)$. The film's response is seen as a capacitance change between the detection plate and the substrate, which is

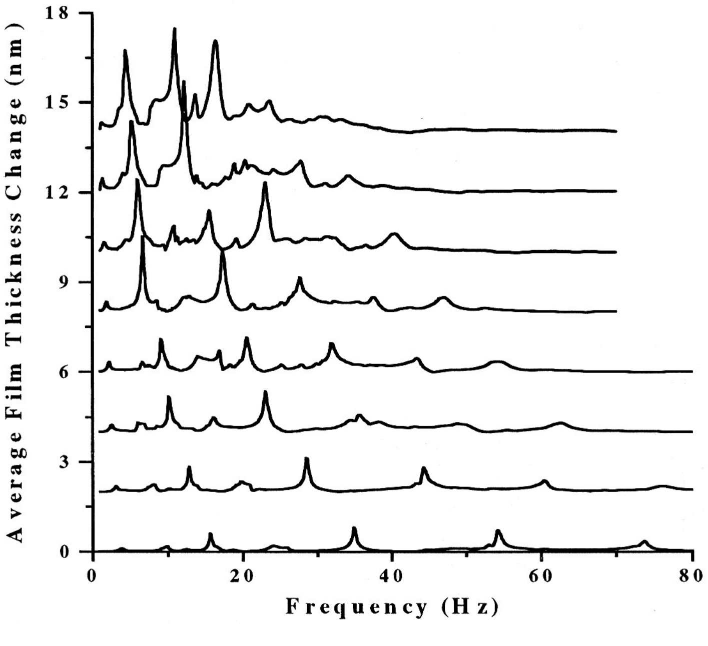

Fig. 2. Third sound resonance spectra at 1.4K for film thicknesses of 85, 78, 72, 66, 61, 56, 49 and 43 nm (from top to bottom).

measured using a symmetric capacitance bridge circuit[13], operating near 3 kHz. The capacitance bridge is well-suited for measuring changes in film thickness at the low frequencies where we observe third sound resonances in saturated films of ^{4}He, and where we anticipate searching for them in films of ^{3}He. With it, we can observe an average film thickness change of 1 pm in 1 sec. Recently, we have used this apparatus to carry out film flow experiments in superfluid ^{3}He down to a temperature of 0.40 mK, with a bridge drive of 8Vp-p at 3 kHz, without seeing a detectable heating of the sample.

The output of the capacitance bridge is then sent to a lock-in amplifier tuned to the drive frequency ν. The output of this lock-in, recorded as a function of the frequency, convolutes the response of two systems to the driving force. In addition to the response of the helium film, the detection plate itself moves in response to the applied voltage, because it is supported by the same superstructure as is the drive plate. In order to account for this motion, we record, as a baseline spectrum, the amplitude and phase of the capacitive response when the cell is empty, and then subtract in quadratures this baseline from each of the spectra taken with the helium film in the resonator.

Fig. 2 shows a set of resonance spectra (after the baseline has been subtracted), taken at 1.4 K for a range of film thicknesses from 43 nm to 85 nm. For each of these spectra, we are able to identify a set of peaks whose frequencies correspond to the zeros of the $J_o(x)$ Bessel function. The quality factor Q of the resonances is sufficiently good to allow us to identify the resonant frequencies to within at least 0.1 Hz. Fig. 3 shows the linear fit of the resonant frequencies to the first five zeros x_{on}.

Using these frequencies and equation (3), we were able to determine the velocity of third sound for the ^{4}He film as a function of film thickness. Fig. 4 shows this result by graphing the square of the third sound velocity vs. the height

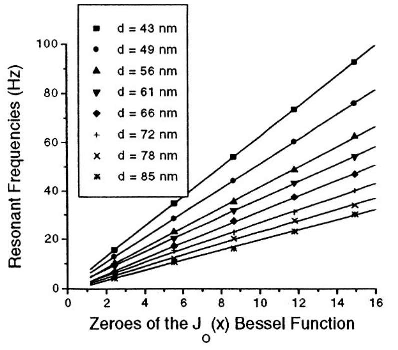

Fig. 3. The resonant frequencies (taken from the peaks corresponding to the first five x_{on}) vs the five x_{on}.

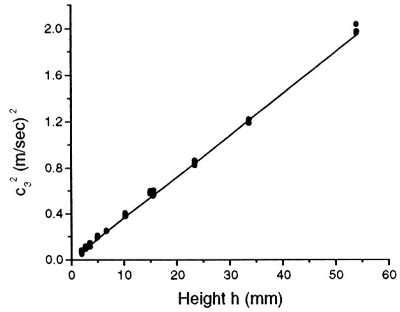

Fig. 4. C_3^2 vs the height from the film to the bulk surface. The line shows the expected relation when the van der Waals exponent takes its thick film value $n = 4$.

h. The data fits well to the expected relation given by equation (1). Note that our Fig. 4 includes additional data that is not shown in Figs. 2 and 3, taken at lower film coverages (26, 31, 36, and 40 nm), and correspondingly larger heights h.

CONCLUSION

We have developed an apparatus which can capacitively generate and detect third sound resonances in saturated films of helium. We have demonstrated that this resonator can measure the velocity of third sound in thick films of superfluid ^{4}He. The resonator can be cooled to sub-mK temperatures, and we intend to use it to look for third sound resonances in saturated films of ^{3}He.

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